

Forme indeterminate II

Approfondimento 1

A. Forme indeterminate del tipo $\infty - \infty$ che sono differenze di funzioni irrazionali

Nozioni da applicare nei calcoli

$$(\sqrt{a} - \sqrt{b}) \cdot (\sqrt{a} + \sqrt{b}) = a - b \Rightarrow (\sqrt{a} - \sqrt{b}) = \frac{a - b}{\sqrt{a} + \sqrt{b}} \text{ con } a > 0 \text{ e } b > 0$$

$$(\sqrt{a} - b) \cdot (\sqrt{a} + b) = a - b^2 \Rightarrow (\sqrt{a} - b) = \frac{a - b^2}{\sqrt{a} + b} \text{ con } a > 0$$

Esempi di calcoli svolti

ESEMPIO 1

$$\lim_{x \rightarrow +\infty} \left(\boxed{\sqrt{x+1}} - \boxed{\sqrt{x}} \right)$$

Diagram illustrating the initial limit expression. The term $\sqrt{x+1}$ is enclosed in a blue box with a blue arrow pointing to a blue $+\infty$ above it. The term $\sqrt{x}$ is enclosed in a red box with a red arrow pointing to a red $+\infty$ above it.

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x+1} - \sqrt{x} \right) = \lim_{x \rightarrow +\infty} \frac{x+1-x}{\boxed{\sqrt{x+1} + \sqrt{x}}} = 0$$

Diagram illustrating the rationalization step. The denominator $\sqrt{x+1} + \sqrt{x}$ is enclosed in a red box with a red arrow pointing to a red $+\infty$ below it.

$$\boxed{\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a} + \sqrt{b}}}$$

Esempi di calcoli svolti

ESEMPIO 2

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 4} - x \right)$$

Diagram illustrating the limit calculation. A blue box highlights the expression $\sqrt{x^2 + 4}$, with a blue arrow pointing to $+\infty$. A red box highlights the expression x , with a red arrow pointing to $+\infty$.

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 4} - x \right) = \lim_{x \rightarrow +\infty} \frac{x^2 + 4 - x^2}{\left(\sqrt{x^2 + 4} + x \right)} = 0$$

Diagram illustrating the limit calculation. A red box highlights the denominator $\left(\sqrt{x^2 + 4} + x \right)$, with a red arrow pointing to $+\infty$.

$$\left(\sqrt{a} - b \right) = \frac{a - b^2}{\sqrt{a} + b}$$

B. Forme indeterminate del tipo ∞/∞ che sono quozienti di funzioni irrazionali

Nozioni da applicare nei calcoli

$$\sqrt{a} \text{ esiste solo se } a \geq 0$$

$$\sqrt{a} = a^{\frac{1}{2}}, \text{ con } a \geq 0$$

$$\frac{a}{\sqrt{a}} = \sqrt{a}, \text{ con } a > 0$$

$$\frac{\sqrt{a}}{a} = \frac{1}{\sqrt{a}}, \text{ con } a > 0$$

$$(\sqrt{a})^2 = a, \text{ con } a \geq 0$$

$$\sqrt{a^2} = |a| \text{ per qualunque } a \text{ reale}$$

$$|a| = \begin{cases} a, & \text{se } a \geq 0 \\ -a, & \text{se } a < 0 \end{cases}$$

$$\frac{|a|}{a} = \begin{cases} 1, & \text{se } a > 0 \\ -1, & \text{se } a < 0 \end{cases}$$

Esempi di calcoli svolti

ESEMPIO 3

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^2 + 3}}{x}$$

Diagram showing the limit expression with annotations: a blue box around the numerator $\sqrt{2x^2 + 3}$ with an arrow pointing to $+\infty$, and a red box around the denominator x with an arrow pointing to $+\infty$.

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^2 + 3}}{x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 \left(2 + \frac{3}{x^2}\right)}}{x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2} \cdot \sqrt{2 + \frac{3}{x^2}}}{x} = \lim_{x \rightarrow +\infty} \frac{|x|}{x} \cdot \sqrt{2 + \frac{3}{x^2}} = \sqrt{2}$$

Diagram showing the step-by-step simplification of the limit. Annotations include: a green arrow pointing from the first expression to the second; a red arrow pointing from the $|x|/x$ term to the box containing the explanation; and a blue arrow pointing from the $3/x^2$ term to the box containing the value 0.

Raccolgo la potenza di x
con esponente massimo

x tende a $+\infty$ perciò, nel
calcolo del limite, ho $x > 0$

Esempi di calcoli svolti

ESEMPIO 4

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 3}}{x} :$$

Diagram showing the initial limit expression. A blue box highlights the numerator $\sqrt{2x^2 + 3}$ with an arrow pointing to $+\infty$. A red box highlights the denominator x with an arrow pointing to $-\infty$.

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 3}}{x} = \lim_{x \rightarrow -\infty} \frac{|x|}{x} \cdot \sqrt{2 + \frac{3}{x^2}} = -\sqrt{2}$$

Diagram showing the algebraic manipulation of the limit. The original expression is equated to the product of two limits. The first limit, $\lim_{x \rightarrow -\infty} \frac{|x|}{x}$, has a red box around $\frac{|x|}{x}$ with an arrow pointing to -1 . The second limit, $\lim_{x \rightarrow -\infty} \sqrt{2 + \frac{3}{x^2}}$, has a blue box around $\frac{3}{x^2}$ with an arrow pointing to 0 . The final result is $-\sqrt{2}$. A green arrow points from the first limit to the text box below.

Raccolgo la potenza di x
con esponente massimo

x tende a $-\infty$ perciò, nel
calcolo del limite, ho $x < 0$

Esempi di calcoli svolti

ESEMPI 4 e 5

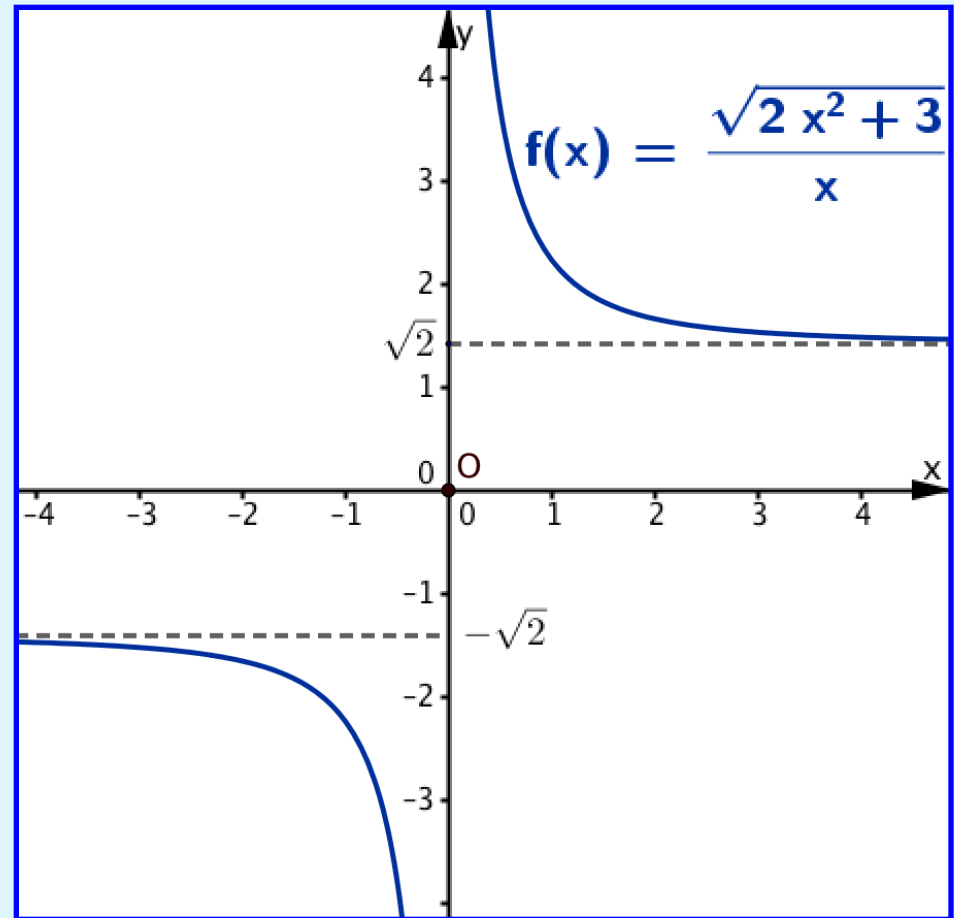
In conclusione ho ottenuto

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^2 + 3}}{x} = \sqrt{2}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 3}}{x} = -\sqrt{2}$$

E quindi

$$\text{Non esiste } \lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 3}}{x}$$



C. Collegamento fra forme $\infty - \infty$ e ∞/∞ con funzioni irrazionali

Esempi di calcoli svolti

Esempio 6

$$\sqrt{a} - \sqrt{b} = \frac{a - b}{\sqrt{a} + \sqrt{b}}$$

$$\lim_{x \rightarrow +\infty} (\sqrt{4x+5} - \sqrt{x}) = \lim_{x \rightarrow +\infty} \frac{4x+5-x}{\sqrt{4x+5} + \sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{3x+5}{\sqrt{4x+5} + \sqrt{x}}$$

$$\lim_{x \rightarrow +\infty} \frac{3x+5}{\sqrt{4x+5} + \sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{x \left(3 + \frac{5}{x} \right)}{\sqrt{x} \left(\sqrt{4 + \frac{5}{x}} + 1 \right)} = \lim_{x \rightarrow +\infty} \sqrt{x} \left(\frac{3 + \frac{5}{x}}{\sqrt{4 + \frac{5}{x}} + 1} \right) = +\infty$$

Raccolgo la potenza di x
con esponente massimo

Esempi di calcoli svolti

Esempio 7

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 4x + 3} - x \right)$$

Diagram showing the initial expression with annotations: a blue box around $\sqrt{x^2 + 4x + 3}$ and a red box around x . A blue arrow points to $+\infty$ above the box, and a red arrow points to $+\infty$ above the box.

$$(\sqrt{a} - b) = \frac{a - b^2}{\sqrt{a} + b}$$

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 + 4x + 3} - x \right) = \lim_{x \rightarrow +\infty} \frac{x^2 + 4x + 3 - x^2}{\sqrt{x^2 + 4x + 3} + x} = \lim_{x \rightarrow +\infty} \frac{x \left(4 + \frac{3}{x} \right)}{x \left(\sqrt{1 + \frac{4}{x} + \frac{3}{x^2}} + 1 \right)} = \frac{4}{2} = 2$$

Diagram showing the algebraic manipulation of the limit. The expression is simplified to $\frac{x(4 + \frac{3}{x})}{x(\sqrt{1 + \frac{4}{x} + \frac{3}{x^2}} + 1)}$. Annotations include: a blue box around $4x+3$ in the numerator, a red box around $\sqrt{x^2 + 4x + 3} + x$ in the denominator, and a blue box around $\frac{3}{x}$ in the numerator. Arrows indicate the limits of these terms: $+\infty$ for $4x+3$, $+\infty$ for the denominator, 0 for $\frac{3}{x}$, and 0 for $\frac{4}{x}$ and $\frac{3}{x^2}$.

Raccolgo la potenza di x
con esponente massimo

x tende a $+\infty$ perciò, nel
calcolo del limite, ho:
 $x > 0$ e quindi $|x| = x$

Forme indeterminate con funzioni irrazionali

Sono limiti impegnativi nei calcoli con carta e penna, non solo quando sono basati sulle proprietà algebriche illustrate in questo approfondimento, ma anche quando sono basati sulle derivate.